Savings Accounts Math - Compounding

P1. The amount of the interest income increases with the increasing invested amount, the level of interest rates and the compounding frequency. Assume you deposit $6,500 (PV, Present Value) into an account paying an interest rate. How much money will be in the account (i.e. FV, Future Value) and how much cumulative interest income will be realized (i.e. FV – PV) after 7 years when:

a. The account pays 2% annual interest compounded annually. Annual compounding is typically used in GIC (Guaranteed Investment Certificates).

b. The account pays 4% annual interest compounded annually. Compare your answer to that in P1a. How much did the cumulative interest income increase when you doubled the interest rate – less than double, exactly double, or more than double? Why?

c. The account pays 4% annual interest compounded semi-annually. Compare your answer to that in P1a. How much did the cumulative interest income change (and did it increase or decrease) when you increased compounding frequency? Why?

d. The account pays 4% annual interest compounded quarterly.

e. The account pays 4% annual interest compounded monthly.

f. The account pays 4% annual interest compounded weekly.

g. The account pays 4% annual interest compounded daily and 7 year period includes 2 leap years (2020 and 2024). What's a leap year? Before you solve the problem, reflect on how leap years may affect compound interest calculations.

h. Account pays 4% annual interest compounded continuously and 7 year period includes 2 leap years (2020 and 2024).

P2. How much money would you need to deposit today at 3% annual interest compounded quarterly to have $12,000 after 6 years?

P3. If you deposit $5,000 into an account paying 2.5% annual interest compounded semi-annually, how long until you have $8,000 in the account?

P4. At 3% annual interest compounded monthly, how long would it take to double your money? At 3% annual interest compounded annually, how long would it take to double your money? Use the rule of 72 to get an estimate, then use logarithms to calculate the exact value (HINT: $interest\ rate \cdot years = 72$).

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Activity Answer Key

A1a. 2% interest, annual compounding

$$\mathrm{FV} = \mathrm{PV} * (1 + \mathrm{r})^{\mathrm{t}}$$
$$\mathrm{FV} = 6500(1 + 0.02)^{7}$$
$$\mathrm{FV} = \$7{,}466.46$$

$$FV - PV = \$7{,}466.46 - \$6{,}500 = \$966.46$$

A1b.4% interest, annual compounding

$$\mathrm{FV} = \mathrm{PV} * (1 + \mathrm{r})^{\mathrm{t}}$$
$$\mathrm{FV} = 6500(1 + 0.04)^{7}$$
$$\mathrm{FV} = \$8{,}553{,}56$$

$$FV - PV = \$8{,}553.56 - \$6{,}500 = \$2{,}053.56$$

Doubling of interest rate results in more than doubling of the cumulative interest income due to compounding effect.

A1c. Semi-annual compounding

$$FV = PV * \left(1 + \frac{r}{n}\right)^{nt}$$
$$FV = 6500\left(1 + \frac{0.04}{2}\right)^{(2*7)}$$

$$FV = 6500(1.02)^{14}$$
$$FV = \$8{,}576.61$$

$$FV - PV = \$8{,}576.61 - \$6{,}500 = \$2{,}076.61$$

Increasing compounding frequency increases the cumulative interest income due to compounding effect.

Note that semi-annual compounding is typically used for zero-coupon bonds (we don't discuss them in the current version of FinStart).

A1d. Quarterly compounding

$$FV = PV * \left(1 + \frac{r}{n}\right)^{nt}$$
$$FV = 6500\left(1 + \frac{0.04}{4}\right)^{(4*7)}$$

$$FV = 6500(1.01)^{28}$$
$$FV = \$8{,}588.39$$

$$FV - PV = \$8{,}588.39 - \$6{,}500 = \$2{,}088.39$$

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A1e. Monthly compounding

$$FV = PV * \left(1 + \frac{r}{n}\right)^{nt}$$
$$FV = 6500\left(1 + \frac{0.04}{12}\right)^{(12*7)}$$

$$FV = 6500(1.00\overline{3})^{84}$$
$$FV = \$8{,}596.34$$

$$FV - PV = \$8{,}596.34 - \$6{,}500 = \$2{,}096.34$$

A1f. Weekly compounding

$$FV = PV * \left(1 + \frac{r}{n}\right)^{nt}$$
$$FV = 6500\left(1 + \frac{0.04}{52}\right)^{(52*7)}$$

$$FV = 6500(1.0007692308)^{364}$$
$$FV = \$8{,}599.42$$

$$FV - PV = \$8{,}599.42 - \$6{,}500 = \$2{,}099.42$$

A1g. Daily compounding, leap year

$$FV = PV * \left(1 + \frac{r}{365}\right)^{365*t+number\ of\ leap\ years}$$
$$FV = 6500\left(1 + \frac{0.04}{365}\right)^{(365*7+2)}$$

$$FV = 6500(1.0007692308)^{364}$$
$$FV = \$8{,}602.10$$

$$FV - PV = \$8{,}602.10 - \$6{,}500 = \$2{,}102.10$$

When Canadian financial institutions calculate interest rate on daily basis they assume that there are 365 days in a year. Thus, the daily interest rate is calculated as annual interest rate *r* divided by 365. Interest rate accrues (a.k.a. compounds) for one extra day during a leap year. See for example [http://www.tdcanadatrust.com/document/PDF/accounts/513782-20191119.pdf.](http://www.tdcanadatrust.com/document/PDF/accounts/513782-20191119.pdf)

In the example above we assume that there are 2 leap years in 7 year period (e.g. 2020 and 2024). Thus, interest rate accrues for the two extra days for the total of 365*7 + 2 days.

When interest does not accumulate daily (as in the examples above) interest rate calculations are based on 366 days in the leap year. Thus, 4% annual interest in a leap year is a slightly better deal that 4% annual interest rate in a non-leap year for savings account that use daily compounding. Daily compounding is typically used for interest earned on saving accounts and interest paid on credit cards.

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A1h.

$$FV = PV * e^{r*\frac{(365*t+number\ of\ leap\ years)}{365}}$$
$$FV = 6500 * e^{\ 0.04*\frac{(365*7+2)}{365}}$$

$$FV = 6500 * e^{0.04*7.0054795}$$
$$FV = \$8{,}602.23$$

$$FV - PV = \$8{,}602.10 - \$6{,}500 = \$2{,}102.10$$

In the example above we assume that there are 2 leap years in 7 year period (e.g. 2020 and 2024). Thus, interest rate accrues for the two extra days for the total of 365*7 + 2 days. When converted to years (365*7 + 2) /365 = 7.0054795 years.

A2.

$$FV = PV * \left(1 + \frac{r}{n}\right)^{nt}$$
$$12000 = PV * \left(1 + \frac{0.03}{4}\right)^{4*6}$$
$$V = \frac{12000}{1.0075^{24}}$$

$$PV = \$10{,}029.98$$

A3. It would take 19 years (18 years and 335 days).

$$FV = PV * \left(1 + \frac{r}{n}\right)^{nt}$$
$$8000 = 5000\left(1 + \frac{0.025}{2}\right)^{2t}$$
$$\frac{8}{5} = 1.0125^{2t}$$
$$t = \frac{\log_{1.0125}\frac{8}{5}}{2}$$
$$t = 0.5 * \frac{\log_{10} 1.6}{\log_{10} 1.0125}$$
$$t = 18.9174028015381$$

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A4. Using the rule of 72, it would take approximately 24 years.

$$interest\ rate\ \%\ \cdot years = 72$$

$$years = \frac{72}{3}$$

With monthly compounding it would take 23 years and 2 months (i.e. 49 days rounded up).

$$Let\ PV = 100, FV = 200$$

$$200 = 100\left(1 + \frac{0.03}{12}\right)^{12t}$$

$$2 = (1.0025)^{12t}$$

$$t = \frac{\log_{1.0025} 2}{12}$$

$$t = \frac{\log_{10} 2}{12 * \log_{10} 1.0025}$$

$$t = 23.1338$$

With annual compounding It would take 23 years and 6 months (i.e. 164 days rounded up).

$$Let\ PV = 100, FV = 200$$

$$200 = 100(1 + 0.03)^{t}$$

$$2 = (1.03)^{t}$$

$$t = \frac{\log_{1.03} 2}{12}$$

$$t = \frac{\log_{10} 2}{12 * \log_{10} 1.03}$$

$$t = 23.4498$$

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